Research Article

Optimization Of Queueing Model

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Abstract: -in thepaper, we are considering the single server queueing system have interdependent arrival of the service processes having bulk service.In this article, we consider that the customers are served k at any instance except when less then k are in the system &ready to provide service at which time customers are served.

Keyword: -Interdependent queueing models, arrival process, service process, waiting line system, mean dependence.

1. OPTIMIZATION M/M[K]/1 QUEUEING MODEL WITH VARYING BATCH SIZE :-

In this type of systems, the interdependence could be induced by considering the dependent structure with parameters λ , μ and $\in$as marginal arrival rate , service rate and mean dependence rate respectively .

Let $P_n(t)$ be the probability when there are n customers in system at time t . The difference – differential equations of above modelmay have written as,

$$P'_n(t) = -(\lambda + \mu - 2 \in)P_n(t) + (\lambda - \in)P_{n-1}(t) + P_{n-k}(t); \quad n \geq 1$$

$$P'_0(t) = -(\lambda - \in)P_0(t) + (\mu - \in)\sum_{i=1}^{k} P_i(t)$$

.... (1)

Let us considerthat, the system achieved the steady state, therefore the transition equations of considered model ar.

$$-(\lambda + \mu - 2 \in)P_n + (\lambda - \in)P_{n-1} + (\mu - \in)P_{n-k} = 0 \quad ; \; n \geq 1$$

$$-(\lambda - \in)P_0 + (\mu - \in)\sum_{i=1}^{k} P_i = 0$$

.... (2)

Applying heuristic arguments of "Gross and Harris" (1974). One can obtain the solution of mentioned equationsas,

$P_n = Cr^n n \geq 0$, $0 < r < 1$ ....(3)

Where r, is the root of equations which lie in (0,1) of the characteristic equation .

$[(\mu - \in)D^{k+1} - (\lambda + \mu - 2 \in)D + (\lambda - \in)]P_n = 0$ .... (4)

Here Drepresents theoperator.

2. MEASURES OF EFFECTIVENESS: -

The probability that the system is empty is,

$P_0 = (1 - r)$ .... (5)

Where r is as given in equation (3).

For different values of $\in$&k, for the given values of λ and μ , we are able to compute P_0 values & are given in table (5.1). The values of $P_0$for the fixed k , $\in$and for varying λ , μmentioned in the table (5.2).

From tables (5.1), (5.2) and equation number---(5), we observe that for fixed of λ , μ and $\in$, the value of P_0 increases with respect to increase in$k$. As the dependence parameter$\in$increases the value of P_0increases for fixed values of λ , μ and k .The value of P_0decreases for fixed values of μ, k and $\in$ as λincreases. As μ increases the valueof P_0 increases for fixed values of the μ , k and dependence parameter$\in$. If the mean dependence rate, is zero then the value of P_0 is also same as in the $M/M^{[K]}/1 - model$.

The average no. of customers in the system can obtained as

$$L = \frac{r}{1-r}$$

.... (6)

and mean number, of customers in the queue are

$$L_q = \frac{r^2}{1-r}$$

.... (7)