# Chapter: 6
# A Polynomial Related to the Hermite Polynomial

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**Abstract**: A new polynomial $B_n(x)$ which is related to Hermite polynomial $H_n(x)$ has been defined. Its various properties such as explicit relation, recurrence relations, generating relations, differential equation to which it satisfies; etc., have been reported.

1. Srivastava (1964) and singh (1964) considered some polynomials related to the laguerre polynomials. With the idea from their papers, here we define a polynomial $B_n(x)$ related to the Hermite polynomial $H_n(x)$ [See Sneddon (1964)]. It seems that $B_n(x)$ and its properties are of very much interest.

The Hermite polynomial $H_n(x)$ is defined for integral values of $n$ and all real values of $x$ by

the identity $$e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{H_n(x)}{n!} t^n. \tag{1.1}$$

Here we consider a set of polynomials $B_n(x)$, such that

$$\sum_{r=0}^{n} \frac{B_r(x) H_{n-r}(x)}{r!(n-r)!} = 0, n \geq 1 \tag{1.2}$$

$$B_0(x) = 1. \tag{1.3}$$

It follows from (1.2) and (1.3) that

$$1 = \sum_{n=0}^{\infty} t^n \sum_{r=0}^{n} \frac{B_r(x) H_{n-r}(x)}{r!(n-r)!}$$

$$= \sum_{r=0}^{\infty} \sum_{n=0}^{\infty} \frac{B_r(x) H_n(x) t^{n+r}}{r!n!}$$

$$= \sum_{r=0}^{\infty} \frac{B_r(x) t^r}{r!} \sum_{n=0}^{\infty} \frac{H_n(x) t^n}{n!}$$

$$= e^{2xt-t^2} \sum_{r=0}^{\infty} \frac{B_r(x) t^r}{r!}.$$